On the non-individuation of absolute motion in Michelson-Morley and Kennedy-Thorndike experiments

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Abstract

In this article I describe in detail why, following Lorentz's ideas, the experiments mentioned in the title yield a negative result. What makes the demonstration particularly interesting, in my opinion, is its broad generality: the arms can have arbitrary directions.

1 Preliminary accounts

To study Michelson's experiment[1] we will focus on outward and return path of a flash of light along an arm, oriented in arbitrary direction. We will work in a genuinely classical context: we assume the speed of light isotropic only in a privileged reference frame (similar to the case of sound, which is isotropic only for an observer at rest relative to the air). We call "*absolute motion*" the motion relative to this privileged reference frame.

Let $\boldsymbol{v}$ be the absolute speed of the interferometer, and $\boldsymbol{l}$ be the initial position of reflecting mirror at the bottom of the arm. Let us suppose that at zero time the mirror is at the origin (so that $l = |\boldsymbol{l}|$ is the length of the arm) and starts the light flash. After a time t the location of the reflecting mirror is $\boldsymbol{l} + \boldsymbol{v}t$ then the condition for the instant when the flash reaches the mirror is

$$|\boldsymbol{l} + \boldsymbol{v}t| = ct \tag{1}$$

By squaring (we will have a spurious solution) we can reorder in this way

$$(1-\beta^2)t^2 - \frac{2l}{c}\xi t - \frac{l^2}{c^2} = 0 \tag{2}$$

[1]I assume as already known the apparatus in its ideal schematization (the reader could consult [RR]), always described in literature, but never in-depth about calculations.

Where ξ is the component of the absolute velocity along the axis defined by the arm expressed as a fraction of the speed of light:

$$\xi = \beta \cos\theta \tag{3}$$

Where $\beta = \frac{v}{c}$ and θ is the angle between the arm and the absolute velocity. Solving for time, we obtain $\frac{l}{c} \cdot \frac{\xi \pm \sqrt{\xi^2 + (1-\beta^2)}}{1-\beta^2}$. The instant we're looking for is obviously positive, then the solution with the minus is spurious. Let $\hat{t}$ denote the instant when the flash of light reaches the reflecting mirror:

$$\hat{t} = \frac{l}{c} \cdot \frac{\xi + \sqrt{\xi^2 + 1 - \beta^2}}{1 - \beta^2} \tag{4}$$

The flash of light reflected starts then from the point $\boldsymbol{l} + \boldsymbol{v}\hat{t}$. We consider $\tilde{t}$ the instant when the light flash meets again the half silver mirror. Clearly this will take place in the point $\boldsymbol{v}\tilde{t}$. The vector that characterizes the return journey is then $\boldsymbol{v}\tilde{t} - (\boldsymbol{l} + \boldsymbol{v}\hat{t})$. Since we are in absolute reference, and light travels simply at speed c, it should be valid

$$|\boldsymbol{v}\tilde{t} - \boldsymbol{l} - \boldsymbol{v}\hat{t}| = c(\tilde{t} - \hat{t}) \tag{5}$$

By squaring we can reorder in this way

$$(1-\beta^2)\tilde{t}^2 + 2\left[\frac{l}{c}\xi - (1-\beta^2)\hat{t}\right]\tilde{t} + \left[(1-\beta^2)\hat{t}^2 - \frac{2l}{c}\xi\hat{t} - \frac{l^2}{c^2}\right] = 0 \tag{6}$$

Where $\hat{t}$ is given by (4). Solving with respect to $\tilde{t}$ we have a spurious solution (the zero one, it is not difficult to understand its physical significance) and a second solution, which is that we are interested in:

$$\tilde{t} = \frac{2l}{c} \cdot \frac{\sqrt{\xi^2 + 1 - \beta^2}}{1 - \beta^2} \tag{7}$$

We can compare it with limit cases where there is an alignment with the absolute motion (see for example [RR, Par. 1.5]). For the arm 1 we have $\xi = \beta$, while for the arm 2 we have $\xi = 0$. We obtain for the time of outward and return journey the values $\frac{2l}{c}\gamma^2$ and $\frac{2l}{c}\gamma$. The time of outward and return journey (and so the number of wavelengths in the paths) depend on the interferometer arms orientation (through ξ), so we should expect a fringe shift when the interferometer is rotated.

2 The fringe shift expected by Michelson

To estimate the shift, we multiply (7) by c, obtaining that the absolute length of outward and return journey is $\frac{2l\sqrt{\xi^2+1-\beta^2}}{1-\beta^2}$. If the frequency of light emission is ν, the wavelength is $\frac{c}{\nu}$, so the number of wavelengths in the journey is

$$N = \frac{2l\nu}{c} \cdot \frac{\sqrt{\xi^2 + 1 - \beta^2}}{1 - \beta^2} \tag{8}$$

Suppose that arm number 1 is initially parallel to the absolute speed (i.e. we have $\xi = \beta$) and that in consequence of the rotation the arm number 2 becomes parallel to absolute motion. Then the variations in the number of the wavelengths we find in an outward and return journey, due to this rotation for the first and second arm, are

$$\Delta N_1 = \frac{2l_1\nu}{c} \cdot \frac{\sqrt{1-\beta^2}-1}{1-\beta^2} \tag{9a}$$

$$\Delta N_2 = \frac{2l_2\nu}{c} \cdot \frac{1-\sqrt{1-\beta^2}}{1-\beta^2} \tag{9b}$$

If the absolute speed is much smaller than the speed of light, the two expressions are approximately

$$\Delta N_1 = \frac{-l_1\beta^2\nu}{c} \tag{10a}$$

$$\Delta N_2 = \frac{l_2\beta^2\nu}{c} \tag{10b}$$

For one arm the number of wavelengths increases, for the other it decreases. To find the number of shifted fringes we should then sum the absolute values, obtaining the well-known result

$$\boxed{\Delta N = \frac{\beta^2\nu}{c}(l_1 + l_2)} \tag{11}$$

3 The Fitzgerald contraction

As everybody knows, Michelson, even after rotating the interferometer at different times of the year, observed no fringe shift. Less well known is that, in order to explain this null result, Fitzgerald (and then Lorentz), supposed that when an object is in absolute motion, its dimensions are contracted along the direction of motion by γ factor. We show here for which reason a similar hypothesis gives an explanation of this remarkable experimental result. Suppose that the arm is oriented in such a way that when it is in an absolute rest it has, with the direction of the future absolute velocity, an absolute angle θ. When the arm moves at absolute velocity $\boldsymbol{v}$, its components, in the transverse and parallel direction to $\boldsymbol{v}$, they are respectively

$$l \sin\theta \tag{12a}$$

$$l \cos\theta\sqrt{1-\beta^2} \tag{12b}$$

Where the $\sqrt{1-\beta^2}$ factor is due to the contraction supposed by Fitzgerald. The absolute length, once the arm moves, is then

$$l' = l\sqrt{1-\beta^2\cos^2\theta} \tag{13}$$

where θ is the absolute angle that the arm would make with the previous absolute speed, if it were stopped at absolute rest. The absolute angle θ' that now (in absolute motion) the arm makes with the absolute motion must satisfy

$$\begin{cases} l' \sin\theta' = l \sin\theta \\ l' \cos\theta' = l \cos\theta \sqrt{1-\beta^2} \end{cases} \tag{14}$$

from which we obtain θ' as a function of θ

$$\theta' = \tan^{-1}\left(\frac{\tan\theta}{\sqrt{1-\beta^2}}\right) \tag{15}$$

The equation (15) we have obtained seems cumbersome but if we are interested in $\cos(\theta')$, it doesn't make particular problems. By exploiting $\cos(\tan^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$ we notice that $\xi' = \beta \cos\theta'$ (i.e. the absolute speed projection along the new arm direction) is

$$\xi' = \beta \sqrt{\frac{1-\beta^2}{1-\beta^2+\tan^2\theta}} \tag{16}$$

On the basis of the equation (7) the time of outward and return journey, considering Fitzgerald contraction is

$$\frac{2l'}{c} \frac{\sqrt{\xi'^2 + 1 - \beta^2}}{1-\beta^2} \tag{17}$$

So in our case,

$$\frac{2l}{c}\sqrt{1-\beta^2\cos^2\theta} \cdot \frac{\sqrt{\beta^2 \cdot \frac{1-\beta^2}{1-\beta^2+\tan^2\theta} + 1 - \beta^2}}{1-\beta^2} \tag{18}$$

Simplifying we obtain $\frac{2l}{c}\gamma$. If ν is the frequency of light emission, the number of wavelengths that are in an outward and return journey is then

$$\frac{2l\nu}{c\sqrt{1-\beta^2}} \tag{19}$$

We have to notice that θ has disappeared, so if we consider seriously Fitzgerald contraction, we should not be surprised if turning the interferometer (even in different moments of the day and of the year), we do not observe any fringe shift. However a dependence on the *magnitude* of the absolute speed remains. It may be exploited to identify the absolute motion through an experiment as that one we describe in the following paragraph.

4 The experiment of Kennedy-Thorndike

Let's suppose to use an interferometer with arms that have different lengths $l_1 \neq l_2$[2] and to observe constantly the interferometer for months, waiting for variations in the magnitude of the absolute speed.

It may in fact be the case that the magnitude of the absolute speed *does not vary* during the year: assuming for simplicity circular the orbit of the earth, it is sufficient to assume that the absolute speed of the solar system is zero, or perpendicular to the ecliptic (I have *never* seen the problem addressed). The absolute speed of the Sun is unknown, so the absence of fringe shift in the Kennedy-Thorndike's experiment, far from being definitive as suggested in [RR], it is not incompatible with Fitzgerald's idea[3] and it is also compatible with the idea of slowing that we will see in a short time.

Anyway, let's pass over this problem and go on. We call t_A and t_B the initial instant and the final instant of a given time interval (during which we assume that the magnitude of the absolute speed of the laboratory changes significantly). At the instant t_i, the arm l_j defines a journey that contains this number of wavelengths (equation (19)):

$$\frac{2 l_j \nu_i}{c\sqrt{1-\beta_i^2}} \tag{20}$$

where β_i is the absolute speed magnitude in the instant t_i (expressed as fraction of light speed) and ν_i is the absolute frequency of light emitter to time t_i[4]. The variation of the number of wavelengths for the paths associated to the arm 1 and to the arm 2, as a result of the variation of the absolute time from t_A to t_B is then

$$\Delta N_1 = \frac{2l_1}{c}\left(\frac{\nu_B}{\sqrt{1-\beta_B^2}} - \frac{\nu_A}{\sqrt{1-\beta_A^2}}\right) \tag{21a}$$

$$\Delta N_2 = \frac{2l_2}{c}\left(\frac{\nu_B}{\sqrt{1-\beta_B^2}} - \frac{\nu_A}{\sqrt{1-\beta_A^2}}\right) \tag{21b}$$

If we assume that the absolute speed variation does not influence the light

[2]Obviously, *all* interferometers have different length arms. How much it is necessary that lengths are different so the experiment we are describing works? We will consider this delicate point later.

[3]Unless the apparatus was so sensitive to reveal also the absolute speed variations due to the non zero eccentricity of the orbit or to the motion of rotation of the Eart on itself (as is well known it is tilted with respect to the ecliptic). In the light of the considerations that follow the equation (24), it seems an improbable eventuality.

[4]In this context ν is a constant, but I have preferred to introduce this generalisation (frequency of the light emitter dependent from the instant) from here, to simplify later and raise a suspicion in the reader.

emitter frequency, and only in this case, we can say $\nu_A = \nu_B = \nu$ and write

$$\Delta N_1 = \frac{2l_1\nu}{c}\left(\frac{1}{\sqrt{1-\beta_B^2}} - \frac{1}{\sqrt{1-\beta_A^2}}\right) \tag{22a}$$

$$\Delta N_2 = \frac{2l_2\nu}{c}\left(\frac{1}{\sqrt{1-\beta_B^2}} - \frac{1}{\sqrt{1-\beta_A^2}}\right) \tag{22b}$$

This time, the variations in the number of wavelengths are "in the same direction" (if the number of wavelengths increases/decreases to an arm, increases/ decreases for the other too). However, this does not mean that there is no fringe shift, because the variation in the number of wavelengths is directly proportional to the length of the arm. We have already noted that the rotation of the interferometer no longer plays any role, however, if the arms have different lengths, as a result of the variation of the absolute speed the two arms change differently the number of wavelengths contained in the paths. Since the variation in the number of wavelengths is in the same direction, this time to find the number of fringes shifted we must take the absolute value of the difference:

$$\Delta N = \frac{2\Delta l\nu}{c}\left|\frac{1}{\sqrt{1-\beta_B^2}} - \frac{1}{\sqrt{1-\beta_A^2}}\right| \tag{23}$$

where $\Delta l = |l_1 - l_2|$ is the difference of arms length. If $\beta \ll 1$ we have

$$\boxed{\Delta N = \frac{\Delta l\nu}{c}\left|\beta_B^2 - \beta_A^2\right|} \tag{24}$$

Because of light coherence problems, that I have not raised in this article, the first solution that comes to mind to make experimentally accessible the effect, that is increase the difference in length between the arms, was not feasible[5]. Fortunately, it is experimentally accessible even the shift of a small fraction of a fringe (even a thousandth). This makes it possible (assuming optimistically $\left|\beta_B^2 - \beta_A^2\right| \approx 10^{-8}$) detect the effect using visible light, and interferometers whose arms differ in length just a few centimetres (just what it was necessary to preserve the coherence).

The (24) is reported in [RR], but the author omits even mentioning what I explain in the next paragraph. In my view, this is a wrong choice (we could call it even dishonesty) but we must acknowledge Resnick's merit of *raising* interesting problems that are usually ignored in modern physics books[6].

[5]This is a problem of nature completely different from those that I've described here, unessential in relation to what I wanted to prove. The fact is that in nature there is not a perfect source of light, which emits a wave train of infinite duration, monochromatic, coherent, eternal. The reader will easily convince himself that this can create problems when trying to study the interference between waves emitted, albeit from the same source, in times too far apart. I suppose that with modern lasers the coherence problems are less thorny.

[6]The only book that I know that really deals with these problems is [TW] but it still does not report the enlightening (and in my partisan opinion, delightful) demonstration that I elaborated here.

5 The slowing down of processes due to absolute motion

Let us assume that the absolute speed not only generates the contraction of Fitzgerald, but affects the frequency of the emitter, decreasing it by a factor $\sqrt{1-\beta^2}$. If so, at the absolute instants t_A and t_B the emitter emits with frequency $\nu_A = \nu\sqrt{1-\beta_A^2}$ and $\nu_B = \nu\sqrt{1-\beta_B^2}$ where ν is the natural frequency (the absolute frequency of the emitter at absolute rest). We cannot reveal this slowdown with a simple measurement of time because it affects all physical processes. If things are so, the change in number of wavelengths, on a single path, is zero (consider (21)), whatever is the manner in which we rotate the interferometer or we do vary its speed. This explains the absence of fringe shift in the experiment of Kennedy-Thorndike too.

6 Conclusions

But does absolute space exist or not? I invite the reader to reflect on the following wise assertion of Max Born

> No concept or statement that is not susceptible to experimental verification must find its place in a physical theory

And also

> A concept is inherent in the reality to physics when it is possible to detect, by experimental observation, the existence of some phenomena corresponding to it.

Any concept of physics must be connected to experience. This link can perhaps be indirect, it can take place through a mental experiment not yet made or even impossible for practical reasons. It can even be a link of very mysterious nature (as is the case of the de Broglie relations). But such a connection with the experiment *must* exist. Anyone who does not subscribe to these statements, very simply, has no idea what physics *is*. All this of course does not mean that there is something bad in *assuming* the existence of something which apparently is experimentally inaccessible, to study the possibility of revealing it.

In conclusion, in the absence of an operative procedure that allows to locate it (and I do not feel certain to exclude a priori that such a procedure may exist, I myself speculated about an experiment elsewhere) the concept of absolute space is nonsense. Even if such a procedure were theorized, the absolute space would remain a purely fictitious entity up to a possible experimental confirmation.

References

[RR] Robert Resnick (1979). *Introduzione alla relatività ristretta*. Casa Editrice Ambrosiana.

[TW] Edwin Taylor, John Archibald Wheeler (1992). *Spacetime Physics*. Freeman.

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